Indian Statistical Institute

Midterm Examination 2025-2026

Analysis of Several Variables, B.Math Second Year

Time : 3 Hours Date : 08.09.2025 Maximum Marks : 100 Instructor : Jaydeb Sarkar

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(1) (15 marks) Let $f = g(u-v, v-w, w-u)$ be a C^1 function. compute the value of
$$\frac{\partial f}{\partial u} + \frac{\partial f}{\partial v} + \frac{\partial f}{\partial w}.$$

(2) (15 marks) Identify all critical points of the function
$$f(x,y) = x^2y - 6y^2 - 3x^2,$$
and then classify each as the location of a local maximum, local minimum, or saddle point.

(3) (15 marks) Let $g : \{x \in \mathbb{R}^2 : \|x\| = 1\} \longrightarrow \mathbb{R}$ be a nonzero continuous function. Suppose $g(0,1) = g(1,0) = 0$ and $g(x) = g(-x)$ for all $\|x\| = 1$. Define
$$f(x) = \begin{cases} \|x\| g\left(\frac{x}{\|x\|}\right) & \text{if } x \neq (0,0) \\ 0 & \text{if } x = (0,0). \end{cases}$$
Prove that f is not differentiable at $(0,0)$.

(4) (15 marks) Define $f : \mathbb{R}^n \to \mathbb{R}$ by
$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^6} & \text{if } x \neq 0 \\ 0 & \text{otherwise.} \end{cases}$$
Which of the following statements are true (justify your answer)?

(a) f is differentiable at $(0,0)$.

(b) f is continuous at $(0,0)$.

(c) All the directional derivatives of f exist at $(0,0)$.

(5) (15 marks) Find all points at which the direction of fastest change of the function
$$f(x,y) = x^2 + y^2 - 2x - 4y,$$
is $\langle 1, 1 \rangle$.

(6) (15 marks) Consider the function $F : \mathbb{R}^3 \to \mathbb{R}$ defined by
$$F(x,y,z) = x + yz^2 + e^z.$$
Prove that there exists a differentiable function f defined in a neighborhood of $(-1,1)$ such that
$$f(-1,1) = 0,$$
and
$$F(x,y,f(x,y)) = 0.$$
Also, compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(-1,1)$.

(7) (20 marks) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a continuously differentiable function. Prove that f cannot be injective.